LOW-ENERGY EFFECTIVE ACTION IN EXTENDED SUPERSYMMETRIC GAUGE THEORIES.

A.T. BANIN
Institute of Mathematics, Novosibirsk, 630090, Russia

I.L. BUCHBINDER
Department of Theoretical Physics, Tomsk, 634041, Russia
E-mail: joseph@tspu.edu.ru

N.G. PLETNEV
Institute of Mathematics, Novosibirsk, 630090, Russia
E-mail: pletnev@math.nsc.ru

We briefly review a recent progress in constructing the low-energy effective action in $\mathcal{N} = 2, 4$ super Yang-Mills theories. Using superfield methods we study the one- and two-loop contributions to the effective action in the Coulomb and non-Abelian phases. General structure of low-energy corrections to the effective action is discussed.

1 Introduction

Supersymmetric field theories possess many remarkable properties both in the classical and in the quantum levels. The supersymmetry imposes rigid restrictions on a structure of quantum corrections. In some cases these restrictions can be so strong that they allow one to obtain exact results for the effective action at low energies. In $\mathcal{N} = 1$ SUSY models the supersymmetry requirements lead to the known non-renormalizations theorems (see e.g. [1]) and can provide an exact non-perturbative determination of the chiral potential [2].

It is evident that the more extended supersymmetry presents in the model the more strong restrictions are imposed on the effective action. In $\mathcal{N} = 2$ SYM theories supersymmetry requirements (together with duality) allow to get the exact solution for the holomorphic part of the effective action [3]. In $\mathcal{N} = 4$ SYM theory the supersymmetry and the superconformal invariance provide finiteness of the theory and fix an exact form of the non-holomorphic potential which gives the leading low-energy contributions to effective action in $\mathcal{N} = 2$ vector multiplet sector [4]. Generalization of this non-holomorphic potential is the exact complete low-energy effective action depending on all fields of the $\mathcal{N} = 4$ vector multiplet [5].

We consider the effective action on the base of superfield formulations of extended supersymmetric models in $\mathcal{N} = 1$ superspace and $\mathcal{N} = 2$ harmonic superspace. Use of harmonic superspace formulation gives a possibility to explore manifest $\mathcal{N} = 2$ supersymmetry. However, since the operator techniques leading to supersymmetric generalizations of the Heizenberg-Euler or

the Schwinger effective Lagrangians are still well developed only in $\mathcal{N}=1$ superspace, we use $\mathcal{N}=1$ superspace approach for construction of the effective actions beyond leading low-energy approximation and on non-Abelian background.

2 $\mathcal{N}=4$ SYM effective action: exact low-energy effective action depending on all fields of $\mathcal{N}=4$ vector multiplet and two-loop effective action in sector of $\mathcal{N}=2$ vector multiplet

In this section we briefly review a recent progress in construction of the $\mathcal{N}=4$ SYM low-energy effective action for the Coulomb phase in the framework of the $\mathcal{N}=2$ harmonic superspace formulation [6].

The harmonic superspace approach [6] was successfully used to study the effective action [8], [9], [10], [11], [14]. The main attractive feature of such an approach is the possibility to preserve a manifest $\mathcal{N}=2$ supersymmetry on all steps of quantum calculations. For $\mathcal{N}=2$ SYM models in the harmonic superspace the background field method was developed in the papers [12]. Exploring the hidden $\mathcal{N}=2$ supersymmetry of the $\mathcal{N}=4$ SYM theory formulated in the $\mathcal{N}=2$ harmonic superspace, the non-holomorphic potential can be explicitly completed by the appropriate hypermultiplet-dependent terms to the entire $\mathcal{N}=4$ supersymmetric form. Direct calculation in $\mathcal{N}=2$ harmonical superspace allowed to obtain as the exact form of the non-holomorphic potential [11] as the corresponding hypermultiplet dependent complement [5], [13]

$$\Gamma[\mathcal{W},\bar{\mathcal{W}},q^{+}]=c\int d^{12}z\ \left[\ln(\mathcal{W})\ln(\bar{\mathcal{W}})+\mathcal{L}_q(\mathcal{W},\bar{\mathcal{W}},q^{+})\right]\ , \qquad (1)$$

with function

$$\mathcal{L}_q(\mathcal{W},\bar{\mathcal{W}},q^{+})=\left((X-1)\frac{\ln(X-1)}{X}+[\mathrm{Li}_2(X)-1]\right)\ , \qquad (2)$$

here $X=\left(-\frac{q^{ia}q_{ia}}{\mathcal{W}\bar{\mathcal{W}}}\right)$; q^{ia} is the hypermultiplet superfield (see details of denotations in [5]); $\mathrm{Li}_2(X)$ is the Euler dilogariphm function. The bosonic component of the effective action corresponding to (1, 2) looks like $F^4/(|\phi|^2+f_{ia}\,f^{ia})^2$ where ϕ is the complex scalar from $\mathcal{N}=2$ vector multiplet and f_{ia} are the scalars from hypermultiplet (see the details in [5]). The effective Lagrangian (2) was firstly found on the base of purely algebraic analysis [5] and then reproduced by quantum field theory calculations using $\mathcal{N}=2$ background field method and the harmonic supergraphs technique.

Study of the two-loop structure of the $\mathcal{N}=4$ SYM effective action for $SU(N+1)$ gauge group spontaneously broken down to $SU(N)\times U(1)$ has been undertaken in the work [8] to clarify a possibility to describe D3-branes interactions in the superstring theory in the terms of the effective action in the $\mathcal{N}=4$ SYM theory. In particular, in the large N limit in case of $U(1)$

constant background the $\mathcal{N}=2$ superconformal invariant two-loop contribution to the effective action, containing F^6-term in its component form, has been calculated. It was shown that the two-loop effective action in the $\mathcal{N}=2$ vector multiplet sector includes the following term

$$\Gamma_{(2)} = N^2 g^2 \frac{1}{3\cdot 16(4\pi)^4}\int d^{12}z\,(\frac{1}{\bar{\mathcal{W}}^2}\ln\frac{\mathcal{W}}{\mu}\mathcal{D}^4\ln\frac{\mathcal{W}}{\mu}+h.c.) \qquad (3)$$

Namely this functional leads to F^6 term in components. It was proved that both the coefficient at one-loop F^4 term and the coefficient at two-loop F^6 term in $\mathcal{N}=4$ SYM effective action exactly correspond to the corresponding coefficients of the Born-Infeld action expansion in the supergravity background (see the details in Ref. [9] for the one-loop effective action and in Ref. [8] for two-loop effective action). It should be pointed out the new covariant approach to study of one- and two-loop contributions to superfield effective action for $\mathcal{N}=2,4$ SYM theories [14].

3 The one-loop effective action in $\mathcal{N}=2,4$ SYM theories beyond leading low-energy approximation

In this section we briefly review a recent progress in studying the one-loop $\mathcal{N}=2$ SYM theory for Abelian and non-Abelian backgrounds and for $\mathcal{N}=4$ SYM effective action beyond of leading low-energy approximation [15], [16], [17].

We consider a hypermultiplet model coupled to external Abelian $\mathcal{N}=2$ vector multiplet using $\mathcal{N}=1$ superfield formulation and study the induced effective action for $\mathcal{N}=2$ vector multiplet. Non-holomorphic contributions to the effective action are written as a sum of three terms. First of these terms is

$$(\Gamma_{W\bar{W}})_{\rm fin} = \frac{1}{(4\pi)^2}\int d^8z\int_0^\infty dt\, t\mathrm{e}^{-t}\frac{W^2\bar{W}^2}{(\Phi\bar{\Phi})^2}\zeta(t\bar{\Psi},t\Psi)\;, \qquad (4)$$

where the function $\zeta(x,y)$ was defined in [15] and quantities Ψ, $\bar{\Psi}$ are scalars with respect to $\mathcal{N}=1$ superconformal group.

The other two terms are obtained one from another by the replacement $\left(\Gamma^+_{\Phi\bar{\Phi}}\right)_{\rm fin} = \left(\Gamma^-_{\Phi\bar{\Phi}}\right)_{\rm fin}(\Psi\leftrightarrow\bar{\Psi})$ and

$$\begin{aligned}\left(\Gamma^-_{\Phi\bar{\Phi}}\right)_{\rm fin} &= \frac{1}{4(4\pi)^2}\int d^8z\int_0^\infty \frac{dt}{t^2}\mathrm{e}^{-t}\Phi\bar{\Phi}\,\xi(t\bar{\Psi},t\Psi) - \\ &- \frac{1}{12(4\pi)^2}\int d^8z\int_0^\infty dt\, t\mathrm{e}^{-t}\frac{W^2\bar{W}^2}{(\Phi\bar{\Phi})^2}\,\lambda(t\bar{\Psi},t\Psi)\tau(t\bar{\Psi},t\Psi), \end{aligned}\qquad (5)$$

where $\lambda(x,y)$, $\xi(x,y)$, $\tau(x,y)$ are some functions found in [15]. One can show that the functionals (4, 5) can be rewritten in manifestly $\mathcal{N}=2$ superconformal invariant form.

Now we consider a structure of the effective action of $\mathcal{N}=2$ SYM model in a non-Abelian phase. We formulate the model in $\mathcal{N}=1$ superspace, use the background field method and impose the gauge-fixing conditions depending on the gauge parameters α, λ and $\bar{\lambda}$ (see the details in [16], [18]).

The gauge-dependent contribution is concentrated in the non-holomorphic potential $\mathcal{H}$ and can be found at any fixed choice of gauge parameters. For the Landau-DeWitt gauge, i.e. then $\alpha=0$, $\lambda=\bar{\lambda}=1$ we obtain [16]

$$2(4\pi)^2\mathcal{H}=\ln(2)\ln(1-s^2)\,+\tfrac{1}{\sqrt{2}}\ln\left(\tfrac{\sqrt{2}-1}{\sqrt{2}+1}\right)\ln(1-s^2)-\mathrm{Li}_2\left(\tfrac{s^2}{2}\right)+$$
$$+\tfrac{\sqrt{2}-1}{\sqrt{2}}\left[\mathrm{Li}_2\left(\tfrac{s-1}{\sqrt{2}-1}\right)+\mathrm{Li}_2\left(-\tfrac{s+1}{\sqrt{2}-1}\right)\right]+\tfrac{\sqrt{2}+1}{\sqrt{2}}\left[\mathrm{Li}_2\left(\tfrac{s+1}{\sqrt{2}+1}\right)+\mathrm{Li}_2\left(\tfrac{1-s}{\sqrt{2}+1}\right)\right], \tag{6}$$

where the notations $s^2=1-\frac{\Phi^2\bar{\Phi}^2}{(\Phi\bar{\Phi})^2}<0$, $t=\frac{\Phi\bar{\Phi}}{\sqrt{\Phi^2\bar{\Phi}^2}}$ are used; $\mathrm{Li}_2(X)$ is the Euler dilogariphm function. As we see, the form of non-holomorphic potential, in general, depends on a gauge choice. This fact can lead to the ambiguous in derivative expansion in non-Abelian phase. Analogous problem also arises when one defines non-Abelian Born-Infeld action [7].

Now we consider a problem of the hypermultiplet completion to the next-to-leading terms $F^8, F^{10}, ...$ for $\mathcal{N}=4$ SYM theory [17]. Our aim is to develop a systematic procedure allowing to construct an expansion of the one-loop effective action in a power series of Abelian strength F. It was shown [9], [17] that the one-loop contribution can be written as a power expansion of dimensionless combinations $\bar{\Psi}^2=\frac{1}{M^2}\nabla^2W^2$, $\Psi^2=\frac{1}{M^2}\bar{\nabla}^2\bar{W}^2$. The quantity M depends on the chiral fields, which contain scalar fields from the $\mathcal{N}=2$ vector multiplet and the hypermultiplet. In the constant field approximation this expansion is summed to the following expression for the whole one-loop effective action (see details in [9]):

$$\Gamma=\frac{1}{8\pi^2}\int \mathrm{d}^8z\int_0^\infty \mathrm{d}t\,t\,\mathrm{e}^{-t}\frac{W^2\bar{W}^2}{M^2}\,\omega(t\Psi,t\bar{\Psi})\ , \tag{7}$$

where function ω was defined in [8]. The difference between the effective actions with and without the hypermultiplet background fields hides in the quantity M [17]. The expansion of the function ω in power of $\Psi,\bar{\Psi}$ leads to the the series for the effective action (7):

$$\Gamma=\Gamma_{(0)}+\Gamma_{(2)}+\Gamma_{(3)}+\cdots,\quad \Gamma_{(n)}\sim\sum_{m+l=n}c_{m,l}\Psi^{2m}\bar{\Psi}^{2l}\ . \tag{8}$$

In the bosonic sector, this expansion corresponds to expansion in powers of the strength F, namely $\Gamma_{(n)}\sim F^{4+2n}/M^{2+2n}$. The calculations of $\Gamma_{(0)}$ lead to the expression, which was firstly found in [5], [13]. The $\mathcal{N}=2$ form of next term ($\sim F^8$) in the series (8) is reconstructed to the following expression for $\Gamma_{(2)}$:

$$\Gamma_{(2)}=\tfrac{1}{2\cdot5!(4\pi)^2}\int d^{12}z\boldsymbol{\Psi^2}\boldsymbol{\bar{\Psi}^2}\big(\tfrac{1}{(1-X)^2}+\tfrac{4}{(1-X)}+$$

$$+\frac{6X-4}{X^3}\ln(1-X)+4\frac{X-1}{X^2}) \ , \tag{9}$$

here $\mathbf{\Psi}^2 = \frac{1}{\mathcal{W}^2}\bar{D}^4\ln\bar{\mathcal{W}}$. This relation defines $\mathcal{N}=2$ superfield form of F^8 contribution to the effective action depending on all fields of $\mathcal{N}=4$ vector multiplet. Moreover, in the paper of Refs. [17] it was shown that any term in (8) can be written in terms of on–shell $\mathcal{N}=2$ superfields.

4 Conclusion

We have presented the recent results on a structure of the low-energy effective action in extended supersymmetric field theories obtained in our papers [5], [8], [9], [11], [13], [15], [16], [17]. The low-energy effective action has been studied using the the superfield formulations of these theories in standard $\mathcal{N}=1$ superspace and the $\mathcal{N}=2$ harmonic superspace.

Exact low-energy effective action depending on all fields of the $\mathcal{N}=4$ vector multiplet has been constructed for $\mathcal{N}=4$ SYM theory in the Coulomb phase. This result has been firstly obtained by analyzing the invariance of the effective action under hidden $\mathcal{N}=2$ supersymmetry transformations in $\mathcal{N}=2$ harmonic superspace [5] and then reproduced by direct harmonic supergraph calculations [13]. The two-loop effective action in $\mathcal{N}=2$ vector multiplet sector was studied [8] and it was proved that in the t'Hooft limit the coefficient at F^6 term exactly coincides with one in the Born-Infeld action.

The one-loop effective action of various $\mathcal{N}=2$ supersymmetric models including $\mathcal{N}=4$ SYM theory has been studied in the Coulomb and non-Abelian phases taking into account dependence both on the fields of $\mathcal{N}=2$ vector multiplet and hypermultiplet [15], [16]. New $\mathcal{N}=1$ covariant and gauge invariant procedure for finding the effective action was formulated and a derivative expansion was developed on its basis. The concrete results are: the effective action of the $\mathcal{N}=2$ vector multiplet induced by the hypermultiplet, gauge dependence of the effective action on a non-Abelian background in $\mathcal{N}=2$ SYM theory and the one-loop effective action including dependence on all powers of the Abelian strength and all powers of hypermultiplet fields in $\mathcal{N}=4$ SYM theory. In the leading order this action reproduces the complete $\mathcal{N}=4$ supersymmetric low-energy effective action found in [5] and allows to get a higher order correction containing the terms $F^8, F^{10}, \ldots$ with the corresponding hypermultiplet completions.

Acknowledgments

I.L.B is very thankful to the Organizing Committee of QFEXT03 and especially Prof. K. Milton for kind support of his participation at the conference. The work was supported in part by INTAS grant, INTAS-00-00254 and RFBR grant, project No 03-02-16193. I.L.B is grateful to grant for Leading Russian Scientific Schools, project No 1252.2003.2 for partial support. The work of

N.G.P and A.T.B was supported in part by RFBR grant, project No 02-02-17884.

References

1. I.L. Buchbinder, S.M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity* (IOP Publishing, 1995; Revised Edition, 1998)
2. N. Seiberg, *Phys. Lett.* B**318** 469 (1993).
3. N. Seiberg, E. Witten, *Nucl. Phys.* B **426**, 19 (1994); B**431**, 484 (1994).
4. B. de Witt, Grisaru, Rocek, *Phys.Lett.* B**374** 279 (1996); M.Dine, N. Seiberg, *Phys. Lett.* B **409**, 239 (1997).
5. I.L. Buchbinder, E.A. Ivanov, *Phys.Lett.* B**524**, 208 (2002).
6. A. Galperin, E. Ivanov, S. Kalitzyn, V. Ogievetsky, E. Sokachev, *Class. Quant. Grav.* **1**, 469 (1984); A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky, E.S. Sokachev, *Class. Quant. Grav.* **2**, 601 (1985); **2**, 617 (1985); "Harmonic Superspace", Cambridge Univ. Press, (2001).
7. A.A. Tseytlin, hep-th/9908105; I. Chepelev, A.A. Tseytlin, *Nucl. Phys.* B**511**, 629 (1998).
8. I.L. Buchbinder, A.Yu. Petrov, A.A. Tseytlin, *Nucl.Phys.* B**621**, 179 (2002).
9. I.L. Buchbinder, S.M. Kuzenko, A.A. Tseytlin, *Phys. Rev.* D **62**, 045001 (2000).
10. I.L. Buchbinder, E.I. Buchbinder, E.A. Ivanov, S.M. Kuzenko, B.A. Ovrut, *Phys.Lett.* B**412** 309 (1997); E.A. Ivanov, S.V. Ketov, B.M. Zupnik, *Nucl.Phys.* B**509** 53 (1997); E.I. Buchbinder, I.L. Buchbinder, E.A. Ivanov, S.M. Kuzenko, *Mod.Phys.Lett.* A**13** 1071 (1998); I.L. Buchbinder, I.B. Samsonov, *Mod.Phys.Lett.* A**14** 2537 (1999); S. Eremin, E. Ivanov, *Mod.Phys.Lett.* A**15** 1859 (2000); I.L. Buchbinder, A.Yu. Petrov, *Phys.Lett.* B**482** 429 (2000), hep-th/0003265; E.I. Buchbinder, I.L. Buchbinder, E.A. Ivanov, S.M. Kuzenko, B.A. Ovrut, Physics of Particles and Nuclei, **32**, 641 (2001); N.Ohta, H.Yamaguchi, *Phys.Rev.* D**32** 1954 (1985).
11. I. L. Buchbinder, S. M. Kuzenko, *Mod.Phys.Lett.* A**13** 1623 (1998); E.I. Buchbinder, I.L. Buchbinder, S.M. Kuzenko, *Phys. Lett.* B**446**, 216 (1999).
12. I.L. Buchbinder, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut *Phys.Lett.* B**417** 61 (1998); I.L. Buchbinder, S.M. Kuzenko, B.A. Ovrut *Phys.Lett.* B**433** 335 (1998).
13. I.L. Buchbinder, E.A. Ivanov, A.Yu. Petrov, Nucl.Phys. B**653**, 64 (2003).
14. S.M. Kuzenko, I.N. McArthur, JHEP **0305** 015 (2003); JHEP **0310** 029 (2003); *Phys.Lett.* B**522**, 320 (2001); *Phys.Lett.* B**513** 213 (2001); *Phys.Lett.* B**506** 140 (2001);
15. N.G. Pletnev, A.T. Banin, *Phys.Rev.* D**60**, 105017 (1999); A.T. Banin, I.L. Buchbinder, N.G. Pletnev, *Nucl.Phys.* B**598**, 371 2001;

16. A.T. Banin, I.L. Buchbinder, N.G. Pletnev, *Phys.Rev.* D**66**, 045021 (2002);
17. A.T. Banin, I.L. Buchbinder, N.G. Pletnev, *Phys.Rev.* D**68** 065024 (2003).
18. B.A. Ovrut, J. Wess, *Phys.Rev.* D**25**, 409 (1982).